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STATISTICAL METHODS IN ROCK MAGNETISM

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Summary

This paper describes the statistical techniques available to the experimenter in palaeomagnetic work. The theory of these methods is based on an assumed probability distribution of errors. It is shown that the mathematical requirements of this distribution are obeyed by the observations from rock samples which are known to possess a stable magnetization; observations on rocks with unstable magnetization however do not conform to it. A theoretical derivation is given for this probability distribution.

The problem of estimating the mean direction of magnetization of a geological formation has in recent years become a matter of the greatest geophysical interest since it is from such estimates that the position of the pole of the Earth in past geological ages is determined. This problem is largely one of the judicious choice of samples and a procedure is suggested whereby such estimates may be achieved with the greatest sample economy.

1. *Introduction.*—It is now generally believed that measurements of the directions of magnetization in certain rock types may be used to determine the direction of the Earth's magnetic field at the time the rock was formed. The directions of magnetization in samples from the same geological formation never agree exactly, and are often scattered widely round the mean direction. In extreme cases the differences between specimens are as much as 90°, and for this reason it is necessary to treat the results statistically.

Fisher (1) has suggested a probability distribution as a basis for the statistical treatment of these directions. This distribution is described in Section 2. In Section 3 a check is made to see whether this mathematical distribution is in accord with the distributions observed in practice.

The problem of estimating the mean direction of magnetization of a rock formation is of considerable geophysical interest, since it is from such estimates that the ancient pole position is calculated (Creer, Irving, Runcorn (2)). This problem is discussed in Section 4. In those palaeomagnetic studies which have had this end in view, there is no uniformity in the sampling pattern adopted by various workers. Moreover, the accuracy ascribed to results appears to bear little relation to the number of observations or the work involved. An instance is given in Table I, where seven observations from one geological formation gave a more accurate result than 540 observations from another. Clearly, here is a sampling problem which requires close attention.

TABLE I

Rock Formation	No. of sites	No. of specimens measured	Mean magnetic direction Declination	Mean magnetic direction Inclination	Semi-angle of the cone of confidence within which the true mean lies at $P=0{\cdot}95$
Deccan Traps of India (3)	7	7	149	+56	10°
Triassic sand-stones of England (4)	9	540	33	+27	12°

2. *Fisher's distribution.*—The direction of magnetization of a rock specimen is specified by the angles dip I and declination D. Taking x, y and z axes respectively as north, east and downwards, I is the angle between the observed direction and the x–y plane (measured as positive for downward directions) and D is the angle (measured clockwise) between the x-axis and the projection of the direction on the x–y plane. The direction cosines are therefore

$$l=\cos I\cos D, \qquad m=\cos I\sin D, \qquad n=\sin I. \tag{1}$$

If (λ, μ, ν) are the direction cosines of the true mean direction of magnetization, the cosine of the angle ψ between the direction of a specimen and the true direction is given by

$$\cos\psi=\lambda l+\mu m+\nu n. \tag{2}$$

Fisher (1) has suggested that these directions will, when regarded as points on a unit sphere, be distributed over the sphere with probability density

$$\frac{\kappa}{4\pi\sinh\kappa}\exp\{\kappa\cos\psi\}. \tag{3}$$

This means that the proportion of the specimens expected to fall in a small area A, the normal to which makes an angle ψ with the true mean direction, will be given by A times the expression (3). The density (3) is rotationally symmetrical about the true mean direction; that is, the azimuthal angle χ about this axis is distributed uniformly through 360°. The parameter κ determines the dispersion of the points: if $\kappa=0$, they are uniformly distributed, and for large values of κ they cluster about the true mean direction. The constant factor in (3) ensures that the density adds up to unity over the whole sphere.

To explain the significance of κ in numerical terms, the probability that a direction will be observed which makes an angle of ψ_0 or more with the true mean direction, is studied. When κ is greater than 3, this probability is given with good accuracy by the formula

$$\text{Prob}\,(\psi>\psi_0)=\exp\{-\kappa(1-\cos\psi_0)\}. \tag{4}$$

Therefore

$$1-\cos\psi_0=\frac{-\log_e\{\text{Prob}\,(\psi>\psi_0)\}}{\kappa}. \tag{5}$$

This formula is illustrated in Fig. 1.

When ψ_0 is small this may be written more simply as

$$\psi_0\,(\text{radians})=\sqrt{\left\{\frac{-2\log_e[\text{Prob}\,(\psi>\psi_0)]}{\kappa}\right\}}. \tag{6}$$

For example if $P(\psi>\psi_0)=0{\cdot}5$, $0{\cdot}05$, with ψ_0 now in degrees,

$$\psi_0=\frac{67{\cdot}5}{\sqrt{\kappa}},\quad \frac{140}{\sqrt{\kappa}}. \tag{7}$$

The former is analogous to the Probable Error, while the latter, which might be called the 95 per cent Error, is the analogue of the more useful yardstick $1{\cdot}96\sigma$ for normal distributions beyond which will lie only 1/20 of the deviations from the mean.

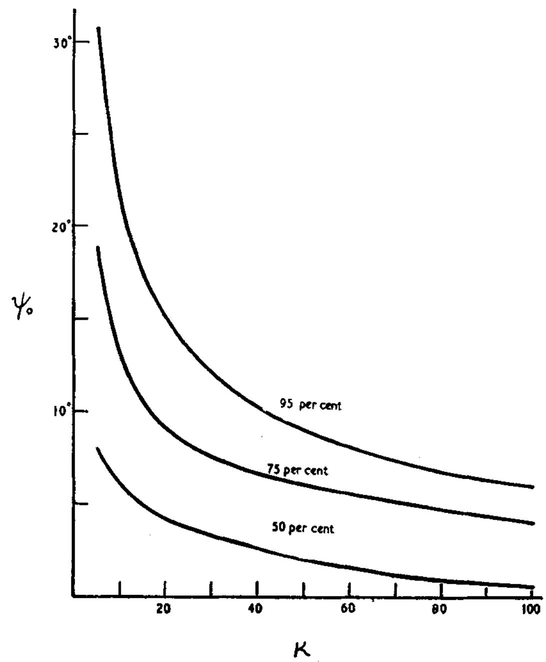

FIG. 1.—*Boundaries for three proportions of distributions with various dispersions. These curves give the semi-vertical angle ψ_0 of the cone around the mean direction which contains 50 per cent ($P(\psi>\psi_0)=0{\cdot}5$), 75 per cent ($P(\psi>\psi_0)=0{\cdot}75$) and 95 per cent ($P(\psi>\psi_0)=0{\cdot}95$) of the observations from distributions with various values of κ.*

The probability distribution (3) has been derived in several physical theories (see e.g. Joos (13)) by considerations of statistical mechanics. If a large number of weakly interacting dipoles of moment m are subject to thermal agitation at absolute temperature T and in a magnetic field of strength H, the proportion of dipoles with energy u is, by the Maxwell–Boltzmann distribution,

$$\text{const.}\times\exp\{-(u/kT)\}.$$

But

$$u=-mH\cos\theta$$

where θ is the angle between the dipole and the field so that the distribution (3) is obtained with

$$\kappa=\frac{mH}{kT}$$

where k here stands for Boltzmann's constant. A fuller discussion of this derivation and its implications will be given in a further paper.

The distribution is also a generalization of a two-dimensional form given by Jeffreys (14).

2.1. *Estimation of parameters.*—Fisher has shown that the best estimate (l, m, n) of the true mean direction (λ, μ, ν) is the vector sum of the individual directions (l_i, m_i, n_i), that is,

$$l = \frac{\Sigma l_i}{R}, \quad m = \frac{\Sigma m_i}{R}, \quad n = \frac{\Sigma n_i}{R} \tag{8}$$

where (l, m, n) are the direction cosines of the mean direction and where

$$R^2 = (\Sigma l_i)^2 + (\Sigma m_i)^2 + (\Sigma n_i)^2. \tag{9}$$

Fisher also shows that the best estimate k of κ is given by, $k > 3$,

$$k = \frac{N-1}{N-R}, \tag{10}$$

where N is the size of the sample.

2.2. *Test of randomness.*—In practice k is never zero, and this may be due to sampling fluctuations, the true value of κ being in fact zero. To distinguish this case Watson (5) has given a statistical test which is based on the following argument. Given a sample of a size N, the length R of the vector resultant will be large if the sample shows a preferred direction and small if it does not. Assuming that there is truly no preferred direction (i.e. $\kappa = 0$), a value R_0, say, may be calculated which will be exceeded by R with any stated probability. Watson has tabulated R_0 for various sample sizes and probabilities 0·05 and 0·01. To carry out the test, it is merely necessary to enter his tables at the row corresponding to the number of specimens in the sample in order to find the value of R_0 which will be exceeded with a given probability in sampling from a population in which $\kappa = 0$. This method is most convenient for testing the randomness of directions of magnetization in conglomerate pebbles, a matter which is of great importance when the stability of the magnetization of rocks is being investigated (Graham (6)).

2.3. *The comparison of direction dispersions.*—The ordinary methods for the comparison of variances may be used to test whether the direction dispersions observed in several populations differ from one another (Watson (7)). Thus, if samples of N_1 and N_2 specimens give dispersion estimates k_1 and k_2 then

$$\frac{k_1}{k_2} = \frac{\text{variance with } 2(N_2 - 1) \text{ degrees of freedom}}{\text{variance with } 2(N_1 - 1) \text{ degrees of freedom}}, \tag{11}$$

assuming the two populations have the same value for κ. This assumption may then be tested since the right-hand side of (11) has the variance-ratio or F-distribution, and values of $F = k_1/k_2$ far from unity suggest strongly that $\kappa_1 \neq \kappa_2$. For several populations, the ratio of the largest to the smallest k may be used to test the hypothesis that κ is constant over the populations; it may be referred to the tables of the maximum F-ratio (Hartley (8)).

2.4. *Tests on mean vectors.*—The first requirement is an estimate of the accuracy of the calculated mean direction (l, m, n). If c is the cosine of the angle between this mean and the true mean (λ, μ, ν), Fisher has shown that this cosine will be less than c with probability P where

$$c = 1 - \frac{N-R}{R}\left\{\left(\frac{1}{P}\right)^{\frac{1}{N-1}} - 1\right\}. \tag{12}$$

In some circumstances it may be easier to use the approximate result that

$$1 - c = -\frac{\log_e P}{kN}. \tag{13}$$

To test (Watson (7)) whether the true mean directions of p populations are identical, the statistic

$$\frac{2(\Sigma N_i - p)}{2(p-1)} \frac{\Sigma R_i - R}{\Sigma N_i - \Sigma R_i} \tag{14}$$

may be referred to the F-ratio tables with $2(p-1)$ and $2(\Sigma N_i - p)$ degrees of freedom. It is supposed that the sample from the ith population contains N_i specimens and has a resultant of length R_i and that R is the length of the vector sum of resultants of the separate samples. Again, large values of the statistic suggest that the assumption of identical true mean directions is false because the algebraic sum of the sample resultants ΣR_i will then be much greater than the length of their vector sum, R.

3. *The fit of observations to Fisher's distribution*

3.1. *The method of fitting.*—Since both κ and (λ, μ, ν) are unknown it is necessary to compare the data with a distribution of the form (3) in which $\kappa = k$ and $\hat{\psi}$, the estimate of ψ, is the angle between the direction (l_i, m_i, n_i) and the average direction of magnetization (l, m, n) of the sample. Thus $\cos\hat{\psi}$ will now be given by

$$\cos\hat{\psi} = l_i l + m_i m + n_i n.$$

To complete the specification of the sample directions the azimuthal angle $\hat{\chi}$ about the axis direction (l, m, n) is required. The values of $\hat{\psi}$ and $\hat{\chi}$ may be determined exactly by spherical trigonometry but it is much quicker, and quite adequate, to find them graphically. The sample directions are plotted on a stereographic projection so that (l, m, n) coincides with the centre point or vertical axis. The angles $\hat{\psi}$ between the individual directions and the mean, and the azimuthal angles $\hat{\chi}$, can then be read off directly.

The angles $\hat{\chi}$ for all the N observations should be equally distributed over the range (0°, 360°). The statistic, χ^2, which can be used to test whether or not this is so, is calculated from the general formula

$$\Sigma \frac{(f_0 - f_e)^2}{f_e} = \Sigma \frac{f_0^2}{f_e} - N \tag{15}$$

and gives a measure of the divergence between observation and expectation. To make the test, the degrees of freedom (d.f.) of χ^2 must be decided. The general rule is

$$\text{d.f.} = (\text{number of classes}) - 1 - (\text{number of constants fitted}). \tag{16}$$

Here only two constants have been fitted because κ is not relevant and only two of (λ, μ, ν) are independent.

To test whether the angles ψ are distributed as in (4), the expected frequency of the $\hat{\psi}$ between the limits ψ_1 and ψ_2 is given by

$$N\{\exp[k(1-\cos\psi_1)] - \exp[k(1-\cos\psi_2)]\}$$

and χ^2 may be used again.

3.2. *Examples of fit.*—The first example is a sample of 70 very fine-grained sandstone specimens taken from the same site (site number A9) in the Diabaig Group of the Torridonian Sandstone Series of N.W. Scotland (Irving and Runcorn (9)). The directions are plotted in Fig. 2. The observed and expected class frequencies for $\hat{\chi}$ and $\hat{\psi}$ are given in Tables II and III and the associated values of χ^2 show that the fit is excellent. The estimate of κ from these data was 55·2. This means that the dispersion is small; in fact, no observation made an angle of greater than 20° with the sample mean vector.

TABLE II

	Range of $\hat{\chi}$ in degrees								χ^2	significance
Frequency	1–45	46–90	91–135	136–180	181–225	226–270	271–315	316–360		
Observed	10	6	8	12	4	9	11	10	2·34	No
Expected	8·75	8·75	8·75	8·75	8·75	8·75	8·75	8·75		

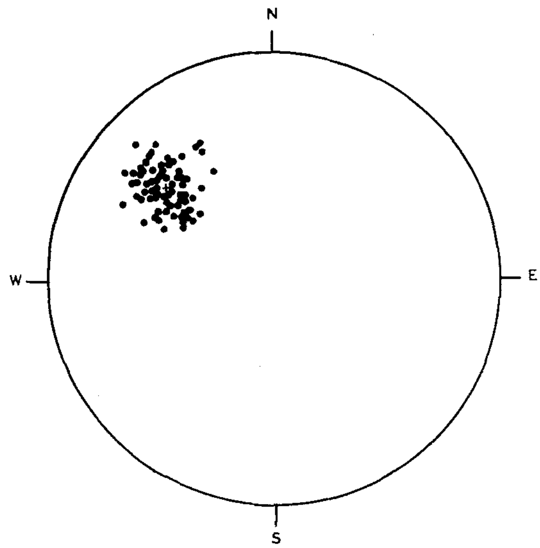

FIG. 2.—*The magnetic directions in samples taken from site* A9 *in the Torridonian Sandstone Series. This illustrates the within-site dispersion* ω *encountered in sedimentary rocks which have a stable magnetization. The mean direction is indicated by a cross. In this figure, and in Figs.* 3 *and* 4, *the north-seeking directions are plotted on stereographic projections with positive magnetic dips indicated by dots, and negative dips by circles. The plane of the projection is in all cases the bedding plane.*

TABLE III

	Range of $\hat{\psi}$ in degrees						χ^2	significance
Frequency	0–3½	3½–6½	6½–9½	9½–12½	12½–15½	15½		
Observed	5	13	15	16	11	10	1·5	No
Expected	7	13·8	16·4	13·9	9·5	9·4		

The second example is a sample of 77 fine, medium, and coarse grained sandstone specimens from the lower part of the Applecross Group of the Torridonian Sandstone (site number B7). The directions used are those measured two months after collection and are plotted in Fig. 3. The observed and expected frequencies for $\hat{\chi}$ and $\hat{\psi}$ are given in Tables IV and V and it is seen that the distribution of $\hat{\chi}$ deviates markedly from Fisher's distribution, whereas the distribution of $\hat{\psi}$ does not.

TABLE IV

	Range of $\hat{\chi}$ in degrees								χ^2	significance
Frequency	1–45	46–90	91–135	136–180	181–225	226–270	271–315	316–360		
Observed	6	7	7	17	4	7	16	13	17·8	Yes
Expected	9·625	9·625	9·625	9·625	9·625	9·625	9·625	9·625		

TABLE V

Frequency	Range of $\hat{\psi}$ in degrees							χ^2	significance
	0–5	5–10	10–15	15–20	20–25	25–30	30		
Observed	3·5	7·5	10	21	12	9	13	4·58	No
Expected	3·16	8·70	12·24	13·17	12·24	9·86	17·63		

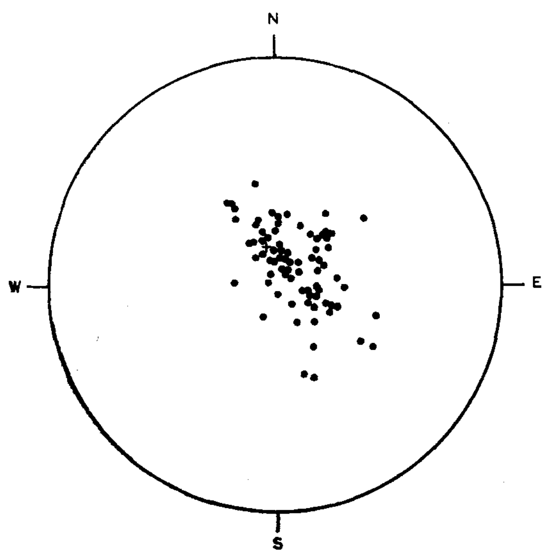

FIG 3.—*The magnetic directions in samples taken from site* B7 *in the Torridonian. This is an example of within-site dispersion in sedimentary rocks with unstable magnetic directions which do not conform to Fisher's distribution. The direction of the present geomagnetic field is indicated by a cross.*

The third example is from the Tasmanian dolerite sills (Irving (**10**)), and the directions considered (Fig. 4) are the mean directions at 30 sites obtained as an average of two samples at each site. The data are insufficient to make a thorough check but Tables VI and VII show that there is no considerable divergence from Fisher's distribution.

TABLE VI

Frequency	Range of $\hat{\chi}$ in degrees				χ^2	significance
	1–90	91–180	181–270	271–360		
Observed	9	7	7	7	0·4	No
Expected	7·5	7·5	7·5	7·5		

TABLE VII

Frequency	Range of $\hat{\psi}$ in degrees				χ^2	significance
	0–5½	5½–9½	9½–15½	15½		
Observed	4	11	9	6	1·7	No
Expected	6	8·5	10·3	5·2		

These examples represent three cases which commonly arise in rock magnetic studies. The first two show the variations in the magnetic directions observed between many samples taken from the same site. The last example illustrates the variations between sites taken from a large rock formation.

In the first example it has been shown by many different tests (Irving and Runcorn (**9**)) that the direction of magnetization was acquired at, or very soon after, the time of deposition and has remained unchanged since the late Pre-Cambrian, over 500×10^6 years ago. The observations obey Fisher's distribution. The specimens in the second example are unstably magnetized since their directions can be changed by storage at an angle to the Earth's magnetic field in a matter of a year. These observations do not fit Fisher's distribution. The rock formation in the third example has been shown to have a stable magnetization (Irving (**10**)), and the mean site resultants again follow Fisher's distribution.

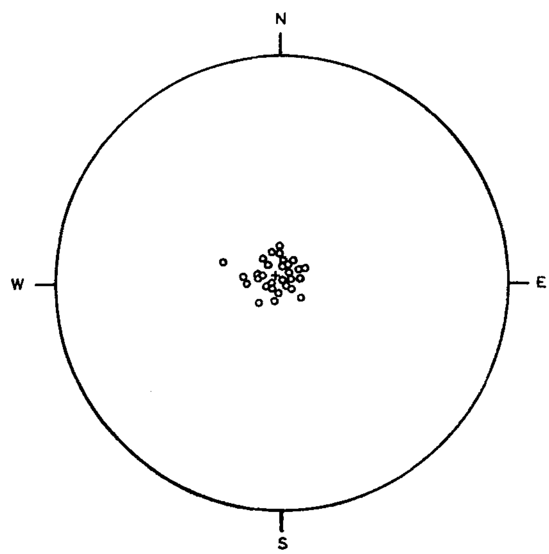

FIG. 4.—*The mean site directions in the Tasmanian dolerite sills. This is illustrative of the between-site variation in a magnetically stable igneous formation. The mean direction is indicated by a cross.*

The lack of fit in the second example may be explained by supposing that two magnetic components are present: one is unstable and directed along the present Earth's field; the other is stable and at a large angle to this field, namely, towards the south-east with positive dip (see Fig. 3). The goodness of fit test may provide a useful means of recognizing those rocks which possess an important unstable component.

4. *The estimation of the mean direction of magnetization of a geological formation.* —In this discussion it is assumed that W_i samples are taken from the ith of B sites. the sites being spaced uniformly through the thickness and areal extent of the formation; and further that the observations within the ith site obey Fisher's distribution with precision ω_i, and the mean site direction varies from site to site with precision β about the overall mean direction. The formation mean direction may be estimated by the direction of the vector resultant of all ($N = \Sigma W_i$) the observations, or, as the direction of the vector resultant of B site mean directions.

4.1. *Igneous rock formations in which ω is constant: uniform magnetization.*—This treatment is approximate, being valid for small dispersions only, that is ω and β large. It is appropriate to observations in basic igneous rocks, notably basalt flows and dolerite sills and dykes.

Estimates $\hat{\omega}$ and $\hat{\beta}$ of ω and β may be found by using the analysis of dispersion given by Watson (7). The observations must first be reduced to give the lengths of the vector resultants at each of the sites, $R_1, R_2, \ldots, R_B$ say, and the length of the resultant of all the $N=\Sigma W_i$ observations, R say. These numbers may then be used to complete this analysis of dispersion table.

Source	Degrees of freedom	Sum of squares	Mean square	Expectations of mean squares
Between sites	$2(B-1)$	$\Sigma R_i - R$	$\dfrac{\Sigma R_i - R}{2(B-1)}$	$\dfrac{1}{2}\left(\dfrac{1}{\omega}+\dfrac{\overline{W}}{\beta}\right)$
Within sites	$2\{\Sigma(W_i-1)\}$	$\Sigma(W_i-R_i)$	$\dfrac{\Sigma(W_i-R_i)}{2\Sigma(W_i-1)}$	$\dfrac{1}{2}\dfrac{1}{\omega}$
Total	$2(N-1)$	$N-R$		

Here
$$\overline{W}=\frac{1}{B-1}\left(N-\frac{\Sigma W_i^2}{N}\right)$$
and is the weighted average of the W_i; if all $W_i=W$, $\overline{W}=W$. The significance of the between-site variation may be judged by an F-test. If the result is significant, estimates $\hat{\omega}$ and $\hat{\beta}$ may be found by equating the mean squares to their expectations and solving the resulting equations; otherwise between-site variation may be ignored, i. e., $\beta \gtrsim \infty$. For example, the data for the Tasmanian dolerites give this table:

Source	Degrees of freedom	Sum of squares	Mean square	Expectations of mean squares
Between sites	58	1·1572	0·01995	$\dfrac{1}{2}\left(\dfrac{1}{w}+\dfrac{2}{\beta}\right)$
Within sites	60	0·5002	0·00834	$\dfrac{1}{2}\dfrac{1}{\omega}$
Total	118	1·6574		

Thus, $F=2{\cdot}39^{**}$, therefore
$$\hat{\omega}=59{\cdot}97 \approx 60, \qquad \hat{\beta}=86{\cdot}13 \approx 86.$$

If the direction of the resultant of all the N observations is used as an estimator of the direction of magnetization of the rock formation, it will be distributed approximately in Fisher's distribution with a precision k given by
$$k=\frac{1}{(\hat{\omega}N)^{-1}+(\hat{\beta}B)^{-1}}. \tag{17}$$
The semi-angle α of the cone of confidence for probability P may then be found from equation (5). For example, the Tasmanian data lead to $k=1500$ so that, for $P=0{\cdot}95=1-1/20$,
$$1-\cos\alpha=\log_e 20/1500, \quad \text{and therefore} \quad \alpha=3^\circ{\cdot}5.$$

If the alternative procedure of using only the site mean directions is followed, a slightly larger cone of confidence is obtained since the data are not so fully used, and this will be particularly so if an unequal number of samples have been taken at each site. The Tasmanian dolerites give, by this method, $\alpha = 3^\circ\cdot 8$. At the present time, the difference between $3^\circ\cdot 5$ and $3^\circ\cdot 8$ in the estimated error is unimportant geophysically. However, it is advisable that authors should give both solutions, since, as palaeomagnetic work progresses, it is inevitable that greater accuracies will be required.

In addition to providing a method by which the error α may be strictly defined, the above analysis allows a maximum estimate of the magnitude of the secular variation of the ancient geomagnetic field to be made from the following argument.

In the Tasmanian dolerites the samples at each site will have been magnetized at approximately the same time, certainly within the same few years. This will not be so between samples from different sites. Consequently the variation within sites may be largely ascribed to experimental error, whereas the between-sites variation will be due to the secular variation of the Earth's magnetic field, and also to an unknown amount of relative block tilting. In the future it may be possible to eliminate the effects of tilting by paying careful attention to the attitude of the strata at each site, and so arrive at a more accurate figure, but in the present example, $\hat{\beta} = 86$ (from (7) 95 per cent Error $= 7^\circ\cdot 3$) can be taken as the maximum estimate of the range of secular variation in the period during which the various members of this extensive complex of sills passed through the Curie temperature. The sills were intruded sometime during the Jurassic or Cretaceous epochs, and the cooling time is at least several thousands of years and probably very much longer (Jaeger and Joplin (**11**), Irving (**10**)).

The question now arises of how many sites should be sampled, and how many observations should be taken at each site, to estimate the formation mean direction with a confidence cone of probability P and semi-angle α. From formula (4) the value of k required for the estimated direction may be found. From an analysis of data from similar formations, or measurements from a preliminary field excursion, estimates of ω and β may be obtained. It is at once evident from (17) that, whatever the values of ω and β, the smallest sample of size N to achieve a given α is made up of a single observation at each of N sites. However, N suitable rock exposures may not be available, in which case as many sites as possible, B say, should be used, and W observations taken at each site so that the relation (17) is satisfied. It is unlikely that the required k will be larger than $\hat{\beta}B$ which is the limiting accuracy with B sites. For example, suppose a 95 per cent cone of confidence with $\alpha = 5^\circ$ is required for a rock formation with ω and β as in the Tasmanian dolerites. Formula (4) gives $k = 786\cdot 28$. If only ten sites are available, equation (17) with $N = WB$ may be solved to give $W = 15\cdot 2$. Thus 16 samples at each site would be required, and the total sample size is $N = 10 \times 16 = 160$. If, however, 30 sites are available, the solution for W is less than unity, so that one sample per site would give a confidence cone of less than 5° half-angle. In this case fewer sites could be used. To find the number required (17) can be solved for B, putting $W = 1$. $B = N = 23$ is then obtained.

In practice it is desirable to take two or more samples at each site to test for gross experimental error and magnetic instability. Evidently, when the overall mean direction of a basic igneous rock formation is required, the most economical use of samples is achieved by taking two samples at each of many sites.

4.2. *Sedimentary rock formation in which ω is variable: non-uniform magnetization.*—Cases are now known in which ω varies from 3 to 2000 at sites in the same sedimentary rock formation (Creer (**12**), Irving and Runcorn (**9**)). In estimating the overall formation direction no satisfactory method is known in which the vector directions of all observations can be taken into account. The site mean vector directions should therefore be used for this purpose. On the assumption that the within- and between-site variations follow Fisher's distribution, then if each is defined with equal accuracy, the site mean vectors will also obey Fisher's distribution. Thus the product $\omega_i W_i$ must be constant at each site. Under these conditions the mean formation direction and its error can be calculated in the usual way.

In order to ensure that $\omega_i W_i$ remains constant it is necessary to regulate the number of samples at each site according to the direction dispersion ω_i. It is a fortunate circumstance that in the cases so far investigated in detail this dispersion depends on grain size, which is a characteristic observable by the collector in the field. From previous data, or from initial sampling, an estimate of ω_i for the rock type at the site may be obtained, and W_i is then decided by successive approximations. Repeated visits to a site may be necessary.

The number of sites required in order to define the overall formation direction with a given accuracy may be obtained from (4), a prior estimate of β, the between-site variation, being obtained from initial sampling.

Reconsidering the results of Table I in the light of the discussion given in this section, it is now apparent why seven measurements from the Deccan Traps give a more accurate mean direction than 540 observations from the Triassic Sandstones of England. In the first case, the optimum use of samples is made by taking one sample at seven widely scattered sites. In the second case, the observations at each site are numerous, and the within-site variation ω is variable. Consequently, the overall mean direction is estimated from nine site resultants, and the individual observations cannot be fully utilized. The variation between sites is greater in the sandstones than in the Deccan lavas giving a larger cone of confidence.

Palaeomagnetic results are now forthcoming from many parts of the world. A uniform procedure both in sampling and analysis greatly facilitates the comparison of results by different authors. If the present methods are used, information on the values of ω and β, which is now being lost, will be recovered and its accumulation will greatly assist future investigations.

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