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A new fold test for palaeomagnetic studies

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SUMMARY

Previous techniques for judging the significance of a palaeomagnetic fold test are either invalid or insufficiently flexible. It is shown that under appropriate circumstances an isolated-observation test may be added to the range of statistical tests used to judge a fold test. A powerful new test is developed based on a test statistic that is sensitive to correlation between the distribution of site-mean directions about the overall mean direction and the tectonic corrections. This test is sufficiently flexible that it should cover most circumstances.

Key words: field tests, fold test, palaeomagnetism, statistics.

1 INTRODUCTION

The fold test (Graham 1949) has, for a long time, been a standard field test in palaeomagnetism for determining whether a magnetization was acquired pre- or post-tectonic folding. For 15 years this test was applied sporadically and without any well-defined criterion for judging its significance until McElhinny (1964) suggested using the ratio of Fisher's precision parameter (Fisher 1953) pre- and post-folding. The test, using McElhinny's criterion, became standard practice wherever possible and thereby created substantially improved discipline in palaeomagnetic investigations. 17 years later McFadden & Jones (1981) showed that McElhinny's criterion is invalid and presented a different criterion for judging a fold test. However, palaeomagnetism continued to evolve and palaeomagnetists were investigating the ancient magnetization in rock formations that had been subjected to complex folding, rather than the relatively simple folding envisaged in the McFadden & Jones test. Consequently there are now several instances of fold tests being applied where the folding has been too complex to lend itself to the McFadden & Jones test. Investigators have therefore reverted to the McElhinny (1964) test even though they recognize it is invalid, but often justifying its use on the basis that the criterion is typically very conservative (McFadden & Jones 1981). Therefore in most cases the conclusions will have been correct when the test was judged to be significant, but undoubtedly there are several instances when the test was incorrectly judged not to be significant.

The intent of this paper is to provide valid criteria for judging the significance of fold tests. The plural is used because it would appear there is no single criterion appropriate for all cases. In some cases it is just a matter of identifying a test that already exists as being the appropriate test. A powerful new test (based on correlation between the distribution of magnetic directions and the tectonic information) is developed here to cater for most of the more complicated situations. Taken together it is hoped that these tests will cover almost all circumstances and, perhaps, be suggestive of an appropriate test for any cases that might not be covered.

Throughout this paper the term 'limb' is used to refer to an area, in a tectonically deformed region, that has suffered no internal deformation. Thus all parts of a limb may be returned to the original undeformed attitude by the application of a single tectonic correction. However, even with a very simple fold, sites close to the axis will be on 'curved' surfaces and so each site may require its own tectonic correction—which is equivalent to each site being on a separate 'limb'. In this paper no restriction is placed on the complexity of the overall deformation (folding). The phrase 'correct relative attitudes' is used to refer to the relative attitudes of the limbs at the time of acquisition of the magnetization, and 'incorrect' to refer to any other relative attitudes.

2 TEST WITH MULTIPLE SITES PER LIMB

If there are multiple palaeomagnetic sites available for each of the fold limbs then it is possible to perform the fold test as laid out by McFadden & Jones (1981), provided the populations of site-mean directions on each limb share a common precision κ.

If there is not a common κ then, as noted in McFadden & Jones (1981), one should investigate a little more closely before applying a fold test. For example, it may be that the folding is more complex than originally recognized, requiring a reassignment of some sites to additional limbs with their own tectonic corrections. If, after further investigation, it is concluded that the tectonics are correctly accounted for, a fold test can still be performed using the correlation test developed later in this paper, provided there is a total of a least five sites reasonably distributed across the limbs. If there are multiple sites on each limb then it is still possible to make use of a test suggested by Watson (1983). To apply this test, if there are r limbs with n_i sites on the ith limb let

$$\left\{\begin{array}{l} X_i = x_{i1} + \cdots + x_{in_i} \\ Y_i = y_{i1} + \cdots + y_{in_i} \\ Z_i = z_{i1} + \cdots + z_{in_i} \end{array}\right\} \tag{1}$$

and

$$R_i = (X_i^2 + Y_i^2 + Z_i^2)^{1/2}, \tag{2}$$

where (x_{ij}, y_{ij}, z_{ij}) are the direction cosines of the jth site-mean direction on the ith limb. Then, using $k_i = (n_i - 1)/(n_i - R_i)$, let

$$S_r = k_1 R_1 + \cdots + k_r R_r, \tag{3}$$

$$\left\{\begin{array}{l} \hat{X} = k_1 X_1 + \cdots + k_r X_r \\ \hat{Y} = k_1 Y_1 + \cdots + k_r Y_r \\ \hat{Z} = k_1 Z_1 + \cdots + k_r Z_r \end{array}\right\}, \tag{4}$$

and

$$R_w = (\hat{X}^2 + \hat{Y}^2 + \hat{Z}^2)^{1/2}. \tag{5}$$

One may now use the test statistic

$$V = 2(S_r - R_w). \tag{6}$$

If the κ_i are all the same ($=\kappa$) then

$$V = 2\kappa\left(\sum R_i - R\right) \tag{7}$$

where R is just the vector sum of all the individual site-mean unit vectors. V is zero if all the site-mean directions are the same, and increases with increasing dispersion of the site-mean directions. Thus the null hypothesis of a common mean direction may be rejected if V is too large. To determine this, V may be compared with the upper $100(1-\alpha)$ per cent point of the $\chi^2_{2(r-1)}$ distribution, provided each of the n_i is at least 25. Unfortunately it is rare to have each n_i this large and, as shown by Watson (1984), V does not have a convenient distribution in small samples. However, with today's wide availability of computing power the inconvenience of a distribution need no longer be an impediment to the use of a statistic, and the question of whether V is 'too large' could be answered by simulation as follows.

(1) Calculate the observed value V_0 of V.

(2) Assuming Fisher distributions (Fisher 1953), under the null hypothesis of a common mean direction, simulate a new set of observations (x_{ij}, y_{ij}, z_{ij}) by choosing $\tau_{\theta ij}$ and $\tau_{\varphi ij}$ from a set of pseudo-random numbers uniformly distributed in the interval [0, 1] and then calculating

$$\left\{\begin{array}{l} \Lambda_{ij} = \dfrac{-\ln\left[\tau_{\theta ij}(1 - e^{-2k_i}) + e^{-2k_i}\right]}{2k_i} \\ \theta_{ij} = 2 \arcsin \sqrt{\Lambda_{ij}} \end{array}\right\}$$

[there are several similar equations for generating pseudo-random variates from a Fisher distribution, but this particular form gives the best performance; see Fisher, Lewis & Willcox (1981)],

$$\varphi_{ij} = 2\pi\tau_{\varphi ij},$$

and

$$\left\{\begin{array}{l} x_{ij} = \cos\theta_{ij}\cos\varphi_{ij} \\ y_{ij} = \cos\theta_{ij}\sin\varphi_{ij} \\ z_{ij} = \sin\theta_{ij} \end{array}\right\}.$$

(3) Calculate the simulated value v_1 of V.

(4) Repeat steps (2) and (3) to obtain 1000 simulated values $v_1, \ldots, v_{1000}$.

(5) Sort the simulated values $v_1, \ldots, v_{1000}$ into ascending order as $V_1, \ldots, V_{1000}$ [in practice this is actually performed concurrent with step (4) by creating a linked-list].

(6) To test at the $100(1-\alpha)$ per cent level, let λ be the largest integer not exceeding $[1000(1-\alpha)+1]$ and reject the null hypothesis if $V_0 > V_\lambda$.

As specified in McFadden & Jones (1981), if the null hypothesis of a common mean direction can be rejected in the *in situ* position, but not in the unfolded position, then this is evidence that the magnetization was acquired before the folding occurred. Conversely, if the null hypothesis can be rejected in the unfolded position but not in the *in situ* position, then this is evidence that the magnetization was acquired after the folding occurred. If the null hypothesis cannot be rejected in either position then this test does not give evidence regarding the age of acquisition of the magnetization. If the null hypothesis can be rejected in both the *in situ* and unfolded positions, but not in some intermediate position, then this is suggestive of the magnetization being syn-deformational (Perroud 1983; McClelland Brown 1983; Schwartz & Van der Voo 1984; Kent & Opdyke 1985; Schmidt & Embleton 1985; Granirer, Burmester & Beck 1986; Miller & Kent 1986; Torsvik *et al.* 1986) or the result of internal strain of a pre-existing magnetization (Facer 1983; Spariosu, Kent & Keppie 1984; Kodama 1986a,b; Van der Pluijm 1987).

3 TEST ON AN ISOLATED OBSERVATION

If there are two limbs with N sites on the first limb and only one site on the second limb (the isolated observation), then it is not possible to estimate the precision on the second limb and so the tests in Section 2 cannot be used. The correlation test developed later in this paper would also be inappropriate. However, if the limbs are in their 'correct' positions, the isolated observation should be just another random observation from the same distribution as the set of observations from the first limb. Thus it is simply a matter of testing whether the isolated observation is discordant with the other observations.

McFadden (1982, equation 14) has shown that if γ_0 is the angle between the isolated observation and the mean direction of the N unit site-mean vectors from the first limb, and if R_N is the resultant length of the vector sum of those N site-mean directions from the first limb, then

$$\cos\gamma_0 = 1 - \frac{(R_N + 1)(N - R_N)}{R_N}\left[\left(\frac{1}{p}\right)^{\frac{1}{(N-1)}} - 1\right] \tag{8}$$

where p is the probability of obtaining an angle γ greater

Table 1. Example of an outlier test used to judge a fold test.

In situ		Unfolded		Tectonic corrections	
Declination	Inclination	Declination	Inclination	Direction of dip	Dip
177.5	10.5	180.9	6.8	259.9	21.3
201.0	8.1	202.0	-0.2	262.8	16.3
196.4	0.1	195.2	-6.6	268.3	20.9
185.1	11.5	187.3	2.2	248.9	19.2
324.3	85.7	196.6	13.9	193.3	75.1
$\gamma_o = 85.4°$		$\gamma_o = 14.3°$			
$\mathrm{Prob}(\gamma > 85.4°) = 0.0005$		$\mathrm{Prob}(\gamma > 14.3°) = 0.204$			

than γ_0. If p is too small (e.g. less than 0.05 for 95 per cent confidence) then the isolated observation may be judged as discordant.

An example is presented in Table 1 from the Mount Eclipse Sandstone of the Ngalia Basin in central Australia (C. Klootwijk, personal communication). Each of the five sites actually has its own tectonic correction, but four of these corrections are very similar and the fifth is quite different. It is therefore sensible to treat the first four limbs as if they were one and test the observation from the fifth to see if it is discordant. Clearly the isolated observation is discordant in the *in situ* position, but not discordant in the unfolded position, which suggests that the magnetization was acquired before the folding occurred.

4 A CORRELATION TEST

Consider now the situation where we have N sites, each on a separate limb with its own tectonic correction. Certainly it is not possible to estimate the between-site precision on any limb and so the above tests cannot be used. Indeed, because of the inability to obtain any estimate of the between-site dispersion that is known to be free of tectonic effects, it appears at first sight to be an impossible task to identify a valid criterion for a fold test. However, even though the McElhinny (1964) test is invalid, it does express in a very vivid way an intuitive recognition that there is useful information present.

In the previous tests the question really being asked is 'is the overall dispersion of the site-mean directions consistent with the observed dispersions of within-limb site-mean directions, assuming that the individual site-mean directions are *random* observations from Fisher distributions sharing a common mean direction?'. This question is then asked independently in the *in situ* and unfolded positions. In the circumstance of a fold test this ignores some powerful information, which is that if we are performing the test with the limbs in relative attitudes different from those in which the magnetization was acquired then the directions of magnetization are not entirely random, but have been moved from their random positions in *known* directions.

Clearly, if the limbs are in the 'correct' relative attitudes (i.e. the same relative attitudes as when the magnetization was acquired) then the distribution of site-mean directions about the overall mean should contain no information about (i.e. should not be correlated with) the tectonics. Conversely, if the limbs are in 'incorrect' relative attitudes then the distribution of site-mean directions will contain information about the tectonics, and if the tectonic deformation has been substantial relative to the natural dispersion of the magnetic vectors then it is possible to detect this information through correlation of the magnetic directions with the tectonics. This leads to an effective and very flexible test.

The flexibility of the test is such that the definition of the test statistic can be tailored to the investigator's particular geological circumstances whilst retaining the same distribution (and therefore critical values) under the null hypothesis. Two definitions of the test statistic are presented here, the choice being based on my perception of their generality and ease of explanation. The distribution of the test statistic is developed within the first definition.

4.1 Definition 1

As shown in Fig. 1, let $\boldsymbol{\mu}$ be the unit vector representing the overall mean direction, and let $\mathbf{m}_i$ be the unit vector representing the ith site-mean direction. Associated with $\mathbf{m}_i$ is the tectonic correction for the limb this site is from. If Fig. 1 is showing the *in situ* $\boldsymbol{\mu}$ and $\mathbf{m}_i$, then the desired tectonic correction is that which would transform $\mathbf{m}_i$ to its unfolded position. Conversely, if Fig. 1 is showing the unfolded directions of $\boldsymbol{\mu}$ and $\mathbf{m}_i$, then the desired tectonic correction is that which would transform $\mathbf{m}_i$ to its *in situ* position. Apply this tectonic correction to $\boldsymbol{\mu}$ to obtain the 'shifted-mean' $\boldsymbol{\tau}_i$. Now define the unit vector $\mathbf{u}_i$ as

$$\mathbf{u}_i = \frac{\boldsymbol{\mu} \times (\mathbf{m}_i \times \boldsymbol{\mu})}{|\boldsymbol{\mu} \times (\mathbf{m}_i \times \boldsymbol{\mu})|}, \tag{9}$$

and the unit vector $\mathbf{v}_i$ as

$$\mathbf{v}_i = \frac{\boldsymbol{\mu} \times (\boldsymbol{\tau}_i \times \boldsymbol{\mu})}{|\boldsymbol{\mu} \times (\boldsymbol{\tau}_i \times \boldsymbol{\mu})|}. \tag{10}$$

$\mathbf{u}_i$ is therefore perpendicular to $\boldsymbol{\mu}$, is in the plane defined by $\boldsymbol{\mu}$ and $\mathbf{m}_i$, and is in the direction from $\boldsymbol{\mu}$ to $\mathbf{m}_i$. Similarly, $\mathbf{v}_i$ is perpendicular to $\boldsymbol{\mu}$, is in the plane defined by $\boldsymbol{\mu}$ and $\boldsymbol{\tau}_i$, and is in the direction from $\boldsymbol{\mu}$ to $\boldsymbol{\tau}_i$. Some may find this easier to visualize by recognizing that if the system were viewed in a frame of reference with $\boldsymbol{\mu}$ perpendicular, then both $\mathbf{u}_i$ *and* $\mathbf{v}_i$ would be horizontal and would, respectively, represent the 'declinations' of $\mathbf{m}_i$ and $\boldsymbol{\tau}_i$ relative to $\boldsymbol{\mu}$. Now

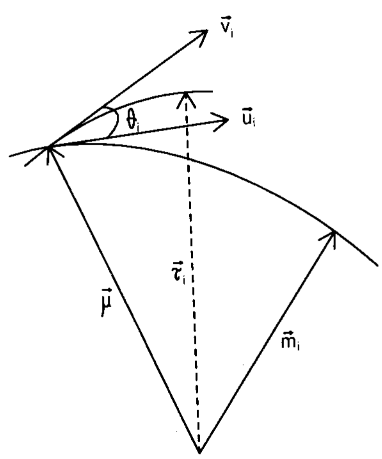

Figure 1. Basic geometry for testing whether there is a correlation between the distribution of magnetic directions and the tectonic corrections. See text for definitions.

define θ_i as the angle between $\mathbf{u}_i$ and $\mathbf{v}_i$, so that

$$\cos\theta_i = \mathbf{u}_i \cdot \mathbf{v}_i. \tag{11}$$

If the limbs are in their 'correct' relative attitudes, $\mathbf{u}_i$ contains no information about (i.e. is not correlated with) $\mathbf{v}_i$. Thus if we make the minimal, and very reasonable, assumption that the magnetic directions are drawn from a population with a uniform azimuthal distribution about the mean (although this does not require a Fisher distribution, it is certainly consistent with that distribution), then θ_i is simply a random observation drawn from a population that is uniformly distributed in the interval $[0, 2\pi]$, and

$$E\{\cos\theta_i\} = 0, \tag{12}$$

where $E\{\cdot\}$ represents the expectation of the variable in the brackets. Conversely, if the limbs are in their 'incorrect' relative attitudes, $\mathbf{u}_i$ will have been 'dragged' towards $\mathbf{v}_i$, so there will be a correlation between $\mathbf{u}_i$ and $\mathbf{v}_i$ giving

$$E\{\cos\theta_i\} > 0. \tag{13}$$

Thus a sensible test statistic for the fold test is

$$\xi = \sum_{i=1}^{N} \cos\theta_i, \tag{14}$$

and we would reject the null hypothesis of no correlation if ξ is too large. The null hypothesis is of course equivalent to the hypothesis that the limbs are in their 'correct' relative attitudes, and rejection of this null hypothesis is equivalent to accepting the alternative hypothesis that the limbs are not in the same relative attitudes as when the magnetization was acquired.

Under the null hypothesis the distribution of ξ is obviously symmetrical and has zero mean. Lord (1948) has derived the distribution function $F_N(\xi)$ of ξ in the form

$$F_N(\xi) - F_N(0) = \frac{1}{\pi}\int_0^{\pi} \frac{\sin \xi t}{t}[J_0(t)]^N\,dt \tag{15}$$

where $J_0(t)$ is the Bessel function of order N. This form is particularly convenient for computation, and the 95 and 99 per cent confidence limits for ξ presented in Table 2 have been calculated using it. Although, for completeness, the table has been presented for values of N as low as 2, it is not recommended that the fold test be performed with $N < 5$. For large N the distribution clearly approximates a normal distribution. Since for $N = 1$ the distribution has zero mean and variance 1/2, the approximating normal distribution has zero mean and variance $N/2$. Thus for large N the 95 and 99 per cent confidence limits for ξ are given by $1.645\sqrt{N/2}$ and $2.362\sqrt{N/2}$ respectively.

It must be recognized that if $\boldsymbol{\mu}$ is beneath the horizontal and the limbs are in their 'incorrect' relative attitudes, then $\mathbf{u}_i$ would have been 'pushed away' from $\mathbf{v}_i$, so $|\xi|$ should be used for comparison with the values in Table 2.

This particular definition of the test statistic has the advantage that it requires only the minimum amount of information—for a simple fold one does not in fact need to know the actual amount of dip of the beds, just the directions of dip to test whether the magnetization could have been acquired with the beds in any given set of relative attitudes. This is because $\boldsymbol{\tau}_i$ moves within a plane containing $\boldsymbol{\mu}$ and so $\mathbf{v}_i$ is independent of the amount of dip. Thus one has the flexibility of performing the test with the beds unfolded to any desired percentage by arbitrarily assigning a dip of say 5°. If there is correlation in both the unfolded and *in situ* positions then this flexibility makes it simple to investigate the possibility of synfolding magnetization by searching for the percentage of unfolding that gives the minimum value of ξ.

Table 2. Critical values of the test statistic ξ.

N	Confidence limit 95%	99%
2	1.697	1.937
3	2.076	2.662
4	2.335	3.180
5	2.609	3.573
6	2.862	3.919
7	3.086	4.253
8	3.298	4.562
9	3.497	4.849
10	3.685	5.120
11	3.865	5.378
12	4.036	5.624
13	4.200	5.860
14	4.358	6.087
15	4.510	6.305
16	4.658	6.516
17	4.801	6.721
18	4.940	6.919
19	5.075	7.112
20	5.207	7.300
21	5.335	7.483
22	5.460	7.662
23	5.583	7.836
24	5.702	8.007
25	5.820	8.174
>25	$1.645\sqrt{N/2}$	$2.326\sqrt{N/2}$

One point of importance here is that because we are seeking correlation with the tectonic corrections, it is critical that a consistent notation be used. In half of the tectonic dips are (randomly) given as negative (i.e., upwards) and the other half as positive, then this will randomly introduce 180° flips in the given directions of dip, and will destroy any correlation. I have therefore adopted the convention that all dips must be positive (i.e. downwards).

The effectiveness of this test has been compared with that of the McElhinny (1964) test by using 5000 random simulations as follows. In each simulation a sample of size N (chosen at random in the range 5–25) was chosen from a Fisher distribution with a mean direction chosen at random (uniform on the surface of the unit sphere) and a precision parameter κ chosen at random in the range 10–50. A set of 'random tectonics' was also chosen for each sample by choosing a random direction of dip (uniform in the interval $[0, 2\pi]$) and a random dip angle (uniform in the interval $[0, 1]$) associated with each direction of magnetization. The set of random dip angles was then scaled so that the maximum dip was equal to 1°. The limbs were then 'folded' according to these tectonics and the significance of the fold test judged according to each of the test criteria. The set of random dip angles was then rescaled so that the maximum dip was equal to 2°, the limbs 'folded' according to these increased tectonic corrections, and the significance of the fold test again judged according to the different criteria. This rescaling of the dip angles was continued until either the maximum dip angle reached 90° (very rare) or the fold test was judged to be significant at the 95 per cent

Table 3. Example data from the top of the Ngalia Basin.

In Situ		Unfolded		Tectonic Corrections	
Declination	Inclination	Declination	Inclination	Direction of Dip	Dip
189.0	56.4	193.4	-11.8	199.1	68.7
173.3	-51.5	180.3	-11.3	12.2	41.5
190.9	43.4	191.0	-6.1	191.2	49.5
186.9	10.7	187.7	-4.3	229.5	20.1
169.3	-15.0	167.0	-13.9	265.0	9.0
181.6	-6.9	180.3	-8.0	264.9	10.0
145.9	-32.9	154.3	-8.1	12.0	32.8
168.1	-47.6	177.5	-16.0	17.9	34.6

$N = 8$

$\xi_1 = 5.958$ $\xi_1 = 2.376$

confidence level according to each of the criteria. The result of this simulation test was that typically the correlation test required only about 40 per cent of the tectonic movement that the McElhinny (1964) test required. In almost all cases the correlation test showed significance at the 99 per cent level of confidence well before the McElhinny test showed significance at the 95 per cent level of confidence.

An example of the use of the correlation test on data from the top of the Ngalia Basin is presented in Table 3 (C. Klootwijk, personal communication). In the *in situ* position ξ_1 (the subscript 1 is used to distinguish this definition of ξ from definition 2 given later) exceeds the critical value at the 99 per cent confidence level, a clear indication that there is correlation between the magnetic directions and the tectonics, so it is unlikely the magnetization was acquired with the limbs in these relative attitudes. Conversely, in the unfolded position, ξ_1 is only 2.376, so there is no reason to reject the hypothesis that the magnetization was acquired before the folding occurred.

A further example, suggesting synfolding magnetization, is presented in Table 4. Here ξ_1 exceeds the critical value at the 99 per cent confidence level in both the *in situ* and the unfolded positions. However, if a partial unfolding is performed, ξ_1 attains a minimum of 0.029 at 44 per cent of unfolding. Thus the evidence suggests that the magnetization was acquired during the folding or was the result of internal strain of a pre-existing magnetization.

4.2 Definition 2

The test statistic given under definition 1 is very effective if the tectonic corrections are such that the angle between the

Table 4. Example suggesting synfolding magnetization.

In Situ		Unfolded		44% unfolded		Tectonic corrections	
Declination	Inclination	Declination	Inclination	Declination	Inclination	Direction of dip	Dip
44.4	-56.2	7.2	-47.5	25.7	-54.6	131.9	29.2
5.5	-58.8	15.2	-76.1	8.0	-66.5	357.3	17.7
69.5	-41.3	287.0	-79.8	63.5	-66.2	76.4	57.3
9.7	-81.7	149.3	-57.5	134.1	-78.8	337.5	39.3
70.9	-58.0	65.6	-69.3	69.1	-63.0	81.3	11.6
101.8	-64.4	226.2	-78.7	121.0	-77.8	85.2	33.5
80.4	-79.9	77.5	-76.3	78.9	-78.3	249.3	3.7
50.2	-52.3	111.7	-74.0	63.9	-64.9	23.5	34.1
40.5	-84.0	110.1	-75.8	87.2	-81.9	315.2	13.4
73.8	-78.4	85.0	-49.9	82.1	-66.0	270.0	28.8
55.7	-56.0	325.8	-63.9	25.2	-68.2	93.9	44.7
339.3	-58.6	22.9	-56.1	359.3	-60.1	266.2	26.9
48.6	-61.5	53.0	-53.5	50.8	-58.0	250.2	8.5

$N = 13$

$\xi_1 = 5.924$ $\xi_1 = 6.676$ $\xi_1 = 0.029$

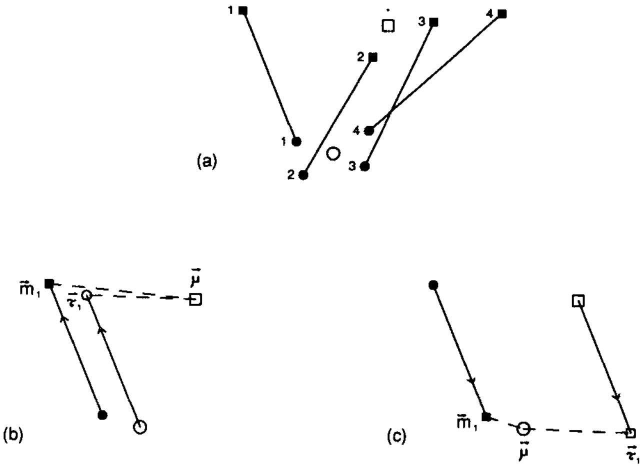

Figure 2. Choice of $\boldsymbol{\tau}_1$ for definition 2 of the test statistic. Circles represent the directions with the limbs in their correct relative positions and squares the directions with the limbs in incorrect relative positions. Solid symbols represent the individual site-mean directions and open symbols the overall mean directions. Smaller open symbols represent the 'shifted-mean'.

in situ overall mean and the unfolded overall mean is small relative to the movements of the individual site-mean directions m_i. However, if the overall mean direction is shifted a large amount (this will happen if, for example, all of the directions of dip fall in a single quadrant) then this definition becomes substantially less effective. There is no problem with the limbs in their correct relative positions, but the choice of $\mathbf{v}_i$ is poor with the limbs in their incorrect relative positions. This can easily be fixed by a more appropriate choice of $\boldsymbol{\tau}_i$, the 'shifted-mean'.

Consider the situation in Fig. 2(a), where a projection of four site-mean directions and their overall mean direction is shown in both the *in situ* and unfolded positions. The solid circles represent the directions of magnetization at the time of acquisition, and the solid squares represent the directions of magnetization with the limbs in their incorrect relative positions. Open symbols represent the respective overall means. It should be clear that definition 1 for the test statistic would show very little correlation for the 'square' directions in Fig. 2(a).

Consider now just observation 1, as shown in Fig. 2(b) and (c). If testing for correlation in the *in situ* position, apply to the overall mean of the unfolded position the rotation that makes $\mathbf{m}_1$ from its unfolded position to its *in situ* position and call this $\boldsymbol{\tau}_1$. Similarly, if testing for correlation in the unfolded position, apply to the overall mean of the *in situ* position the rotation that takes $\mathbf{m}_1$ from its *in situ* position to its unfolded position and call this $\boldsymbol{\tau}_1$. Now, with this new choice of $\boldsymbol{\tau}_1$, define $\mathbf{u}_1$ and $\mathbf{v}_1$ exactly as under definition 1, and θ will, in effect, be the angle between the two dotted lines in each of Fig. 2(b) and (c). If testing with the limbs in their incorrect relative positions, $\mathbf{u}_1$ and $\mathbf{v}_1$ will clearly be correlated, so that θ will be biased towards small values. Conversely, if testing with the limbs in their correct relative positions, θ will be uniformly distributed in $[0, 2\pi]$. Thus we may again choose ξ as defined by equation (14) and, under the null hypothesis, it will have exactly the same distribution as before.

Table 5. Example requiring second definition of ξ.

In Situ		Unfolded		Tectonic Corrections	
Declination	Inclination	Declination	Inclination	Direction of Dip	Dip
353.0	46.0	100.1	41.6	135.0	80.0
355.0	40.0	90.4	40.8	133.0	82.0
7.0	22.0	60.5	33.9	130.0	81.0
13.0	40.0	21.5	43.1	125.0	10.0
354.0	34.0	1.0	42.1	130.0	12.0
4.0	33.0	10.9	38.4	127.0	11.0
354.0	32.0	25.1	55.4	135.0	38.0
358.0	25.0	24.1	47.4	133.0	40.0
0.0	30.0	25.7	47.3	131.0	35.0
34.0	34.0	58.6	32.5	134.0	36.0

N = 10

$\xi_2 = 1.582$ $\quad$ $\xi_2 = 7.411$

An example requiring this second definition is presented in Table 5 (R. Enkin, personal communication). Using the first definition, $\xi_1 = 2.049$, which does not exceed the 95 per cent confidence limit of 3.685, but using the second definition, $\xi_2 = 7.411$, which easily exceeds the 99 per cent confidence limit of 5.120.

This particular definition also works well in situations where the angle between the *in situ* overall mean and the unfolded overall mean is small. However, detection of correlation is most sensitive when one of the positions (*in situ* or unfolded) has the limbs in the correct relative positions. Thus if the magnetization was actually acquired somewhere between these two extremes (i.e. the magnetization is synfolding) the optimal procedure is not immediately obvious. An acceptable approach seems to be that if ξ_2 exceeds (or even approaches) the 95 per cent critical value in both of the extreme positions, a synfolding position should be sought using ξ_1 and then ξ_2 for the *in situ* and fully unfolded positions should be recalculated using this intermediate position as the reference.

4.3 General comments

Two possible (and hopefully reasonably general) definitions of the test statistic have been presented, and it is hoped they will have reasonably general applicability. The particular definition of θ used will of course affect the power of the test, and so it may be that neither of the definitions suggested here is optimal for a given situation. However, the requirements on θ for validity of the test are minimal and so it should usually be possible to identify an optimal definition.

It is important to note that there is no reference to the precision with which any magnetic direction is determined. All that is required is that each of the individual θ_i be a uniform random variate in the interval $[0, 2\pi]$. Thus it is actually possible to use magnetic directions from different hierarchical sampling levels in the one test, greatly enhancing the flexibility of the test. For example, assume there are six limbs, one with 30 sites, one with two sites, and the other four with only one site each. If the correlation test is performed with all 36 site-mean directions then the 30 site-means from the one limb will completely swamp the others, and it is unlikely that any actual correlation will be discernible. However, this can easily be overcome by testing just six directions, each being a limb-mean. It is not critical that one limb-mean comes from 30 sites, one from two sites, while the remaining limb-means are just the individual site-means.

5 CONCLUSION

If the circumstances are appropriate, the significance of a fold test may be judged by testing whether the samples observed on different limbs could have been drawn from distributions sharing a common mean direction. However, in many instances this is not appropriate, and a new test has been developed here based on a correlation between the distribution of the magnetic directions about the overall mean and the tectonic information. This test is very flexible and can be used in most cases.

Programs to perform the tests in this paper may be obtained from BMR via the author.

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