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Angular Dispersion Due to Random Magnetization

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Summary

The angular probability distribution generated by adding a set of randomly oriented vectors to a set of constant parallel vectors has been studied. If one random vector of constant length is added to each constant vector, the resultant angular probability distribution departs widely from the distribution used by Fisher as the basis for palaeomagnetic analysis. However, if n random vectors of equal length are added to each constant vector and if $n > 6$, the resulting angular probability distribution is nearly identical to that proposed by Fisher over most of the range of angular dispersions encountered in palaeomagnetic investigations. Equations are developed from which the mean intensity of random magnetization can be calculated, given the mean total intensity and the angular dispersion. These parameters, in turn, are related to the ratio of random to non-random magnetization for each ferromagnetic mineral grain in a rock specimen.

1. Introduction

Analysis of the natural remanent magnetization of a rock commonly reveals the presence of several components of magnetization, each due to a different magnetizing process occurring at a different time during the rock's history. Even under ideal conditions two components are present, a stable magnetization constant in magnitude and direction from specimen to specimen, and a second component which varies randomly in direction. This second component may be introduced during the collection, the measurement, or the partial demagnetization of the specimen. It may also be present as a fundamental property of natural remanent magnetization, as most processes by which rocks become magnetized do not impart perfectly uniform magnetization to an entire rock formation. Whatever its origin, a randomly oriented component of magnetization is invariably present and produces an angular dispersion in measurements of the remanent magnetization of rocks.

The present investigation is a mathematical analysis of the angular dispersion generated by adding a randomly oriented set of vectors to a set of constant vectors. The inverse problem is also considered: given the angular dispersion of a set of vectors generated in this way and their mean length, what can be said about the lengths of the randomly oriented set? The case is first developed for added vectors of constant length. In a second case, the added vectors are taken to be the resultant of n randomly oriented vectors of equal moment m. This may be taken as a simplified representation of the type of random magnetization suggested by Irving *et al.* (1961), who noted that most processes which produce natural magnetization in rocks succeed in magnetizing only a small fraction of all the magnetic domains present; the remainder are magnetized randomly, hence the vector sum of their individual moments is randomly oriented. The magnitude of this vector sum may assume any value between zero and the algebraic sum of the individual moments, but it has a statistically determined most probable value between these limits.

Comparisons are also made between the angular probability densities generated in this way and the angular probability density proposed by Fisher (1953), which provides the basis for much of the statistical analysis used in palaeomagnetic research. Fisher made the assumption that the vectors of the population from which a sample is drawn are distributed with azimuthal symmetry about a mean polar axis, and further that the angular probability density is given by

$$\Phi_F(\alpha) = (\kappa/4\pi \sinh \kappa)\, e^{\kappa \cos \alpha} \tag{1}$$

where $\Phi_F(\alpha)\, d\chi$ is the proportion of vectors expected to lie within a small cone subtending a solid angle $d\chi$ about a cone axis inclined an angle α from the polar axis. The constant κ is an inverse measure of angular dispersion characteristic of a given population. Giving unit weight to each remanent magnetization vector, Fisher shows that on the basis of N vectors drawn from such a population, the best estimate of the mean is the direction of the vector sum; moreover the best estimate of κ is given by k:

$$k = (N-1)/(N-r) \tag{2}$$

where

$$r = \sum_{i=1}^{N} \cos \alpha_i \tag{3}$$

α_i being the angle between the ith vector and the mean direction.

The mathematical basis for Fisher's assumed probability distribution was thoroughly investigated by Roberts & Ursell (1960). A Brownian distribution, generated by a random walk consisting of many small steps of equal arc length along the surface of a sphere of unit radius, is regarded by Roberts & Ursell as the true analogue of a Gaussian distribution on a plane. Although the equations describing the Brownian and the Fisher distributions are different in form, detailed calculations show that numerical differences between them are small, and these authors conclude that results deduced on the assumption of Fisher's distribution would be little affected by the modifications necessary to bring it to Brownian form. This is fortunate since, as pointed out by Roberts & Ursell, Fisher's distribution is ideally suited for statistical study; other distributions may be theoretically preferable but are analytically cumbersome.

A physical basis for the distribution of Fisher was suggested by Watson & Irving (1957), who noted the formal similarity between equation (1) and the expression $\exp(-mH\cos\alpha/kT)$ derived by Langevin for the proportion of dipoles of moment m which would be inclined at an angle α to a weak magnetic field H when subjected to thermal agitation at temperature T. However the angular dispersion which might be produced in this way is much smaller than that commonly observed in palaeomagnetic studies, and other physical processes are probably responsible for most of the angular dispersion measured in rock specimens (Doell & Cox 1963).

2. Theoretical model

The present analysis employs vectors in three-dimensional space to generate angular dispersion, so that resultant vectors are not constrained to the surface of a unit sphere, as in the analysis of Roberts & Ursell (1960). This approach permits relationships to be established between angular dispersion and variations in vector length. We assume a population of parallel vectors $\mathbf{M}_0$ of constant length M_0 and a second population of randomly oriented vectors $\mathbf{M}$ of varying length M. The population of vectors $\mathbf{J} = \mathbf{M}_0+\mathbf{M}$ generated by adding these two sets is dispersed with azimuthal symmetry about $\mathbf{M}_0$. We proceed to evaluate the probability

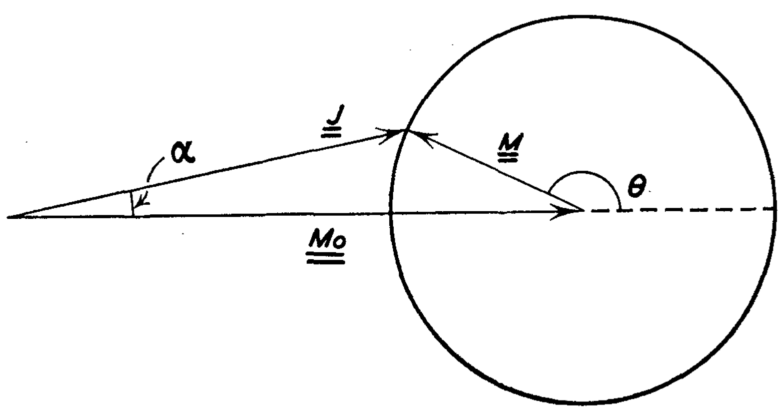

FIG. 1.—Geometry of random vector $\mathbf{M}$ added to constant vector $\mathbf{M}_0$.

distributions for the vector lengths and the angular displacements of the $\mathbf{J}$ vectors.

Let $p(M)$ be the normalized probability distribution of M, assumed known, and let $p(x) = M_0 p(M)$ be the distribution of x, where $x = M/M_0$. Then $p(x)\,dx$ is the probability that M will lie in the interval M to $(M+dM)$ and is also the probability that $\mathbf{J}$ will terminate within a thin spherical shell of radius M and thickness dM about the point O (Figure 1). Taking spherical coordinates about O with θ as the polar angle and $\mathbf{M}_0$ as the direction $\theta = 0$, the proportion of vectors corresponding to an increase $d\theta$ is seen to be $\frac{1}{2}p(x)\sin\theta\,d\theta\,dx$ and the corresponding change in J to be $-M\sin\theta\,(M_0^2+2M_0M\cos\theta+M^2)^{-\frac{1}{2}}\,d\theta$. Writing $z = J/M_0$, the probability distribution for z is:

$$w(z)\,dz = \left[\int z/2xp(x)\,dx\right]dz \qquad (4)$$

if $|1-x| < z < 1+x$, and otherwise $w(z) = 0$, where $w(z)\,dz$ is the probability that J will lie in the interval J to $(J+dJ)$ or equivalently z in the interval z to $(z+dz)$.

The angular probability for the **J** vectors is found by noting that, as θ changes by $d\theta$, the **J** vectors occupy a solid angle $(2\pi \sin\alpha \, d\alpha)$. For $\sin\alpha < x < 1$ there are two values of θ for each value of α, for $x \geqslant 1$ there is only one, and for $x < \sin\alpha$ there are none, resulting in the following angular probability distribution:

$$\Phi(\alpha) = \int_{\sin\alpha}^{1} [(\cos^2\alpha)(x^2-\sin^2\alpha)^{-1/2}+(x^2-\sin^2\alpha)^{1/2}]\, p(x)/2\pi x \, dx +$$

$$+\int_{1}^{\infty} [2\cos\alpha+(\cos^2\alpha)(x^2-\sin^2\alpha)^{-1/2}+(x^2-\sin^2\alpha)^{1/2}]\, p(x)/4\pi x \, dx \qquad (5)$$

where $\Phi(\alpha)\, d\chi$ is the probability that resultant vectors will lie within a small cone subtending a solid angle $d\chi$ about a cone axis inclined an angle α from the direction of $\mathbf{M}_0$.

To describe the angular dispersion of different theoretical probability distributions, an expression is introduced analogous to the quantity δ of Wilson (1959), which was originally defined for a set of N unit vectors as follows:

$$\delta = \cos^{-1}(r/N) = \cos^{-1}(1/N \sum_{i=1}^{N} \cos\alpha_i) \qquad (6)$$

where α_i is the direction between the ith unit vector and the mean. The quantity

$$r = \sum_{i=1}^{N} \cos\alpha_i$$

is first generalized as R to make it applicable to a continuous distribution $\Phi(\alpha)$:

$$R = \int_{0}^{\pi} 2\pi \, \Phi(\alpha) \cos\alpha \sin\alpha \, d\alpha. \qquad (7)$$

The generalized form of δ is then:

$$\Delta = \cos^{-1} R. \qquad (8)$$

An alternative inverse measure of angular dispersion, k of equation (2), has previously been defined for a group of unit vectors. The following generalized form is introduced:

$$K = 1/(1-R). \qquad (9)$$

Like k, K approaches the limit of 1 for a random population and ∞ for a parallel population. This generalization of k is analytically convenient and, when applied to the distribution Φ_F of Fisher, yields a value of K which differs from κ by the quantity $2\kappa^2/(e^{2\kappa}-2\kappa-1)$, which is small for values of $\kappa > 3$. For example, when $\kappa = 4$, K differs from κ by only 0·269 of one per cent, and the difference decreases rapidly for larger values.

The final step in this derivation will be to evaluate these quantities for two probability functions $p(x)$.

3. Random vectors of constant length

We consider first the case of a randomly oriented vector **M** of constant length M_C. Physically this model approximates the occurrence of a constant component of relatively unstable magnetization such as may exist when many ferromagnetic domains have low coercive forces concentrated within a narrow band, with the result that all of the unstable component is magnetized essentially by one event occurring either in nature or during experiments on the specimen. A situation of this type has recently been recognized in a palaeomagnetic study of granites by Currie and others (1963). Introducing $x_C = M_C/M_0$, so that $p(x_C)dx = 1$, the equations of the previous section may be applied directly to give:

$$\Phi_C = (1/2\pi x_C)[(\cos^2\alpha)(x_C^2 - \sin^2\alpha)^{-1/2} + (x_C^2 - \sin^2\alpha)^{1/2}],$$
$$x_C < 1 \text{ and } \sin\alpha < x_C \qquad (10a)$$

$$\Phi_C = 0, \qquad x_C < 1 \text{ and } \sin\alpha > x_C \qquad (10b)$$

$$\Phi_C = (1/4\pi x_C)[2\cos\alpha + (\cos^2\alpha)(x_C^2 - \sin^2\alpha)^{-1/2} + (x_C^2 - \sin^2\alpha)^{1/2}],$$
$$x_C \geqslant 1 \qquad (10c)$$

$$\Delta_C = \cos^{-1}(1 - x_C^{2/3}), \qquad (x_C < 1) \qquad (11a)$$

$$\Delta_C = \cos^{-1}(2/3x_C), \qquad (x_C \geqslant 1) \qquad (11b)$$

and

$$K_C = 3/x_C^2, \qquad (x_C < 1) \qquad (12a)$$

$$K_C = 3x_C/(3x_C - 2), \qquad (x_C \geqslant 1) \qquad (12b)$$

To compare this probability distribution with that of Fisher, the two are plotted in Figure 2 for a wide variety of angular dispersions. Dispersions were made the same in each plot by setting $\kappa = K_C$. The two distributions are seen to differ markedly.

Cumulative differences between these two probability functions were found from the cumulative distribution functions

$$P(\alpha_o) = 2\pi \int_0^{\alpha_o} \Phi(\alpha)\sin\alpha\, d\alpha$$

where $P(\alpha_o)$ is the proportion of all vectors lying within a cone of radius α_o about the mean axis. For the Fisher and constant-vector distribution the respective cumulative functions are

$$P_F = [\exp(\kappa) - \exp(\kappa\cos\alpha_0)]/2\sinh\kappa \qquad (13)$$

$$P_C = 1 - \cos\alpha_0(x_C^2 - \sin^2\alpha_0)^{1/2}/x_C, \qquad x_C < 1$$
$$= \tfrac{1}{2}[1 + (\sin^2\alpha_0)/x_C - \cos\alpha_0(x_C^2 - \sin^2\alpha_0)^{1/2}/x_C], \qquad x_C \geqslant 1. \qquad (14)$$

The two cumulative distributions are substantially different for all angular dispersions. For $\kappa = K_C = 30$, for example, a cone of radius $\alpha_o = 12{\cdot}34^\circ$ contains 50·00 per cent of the vectors of a Φ_F population but only 17·98 per cent of those of a Φ_C population; a cone of radius $\alpha_0 = 18{\cdot}44^\circ$ contains the entire Φ_C population but only 78·55 per cent of the Φ_F population. Thus application of statistical methods based on the distribution Φ_F to a set of samples drawn from a population with the distribution Φ_C would result in erroneous confidence intervals.

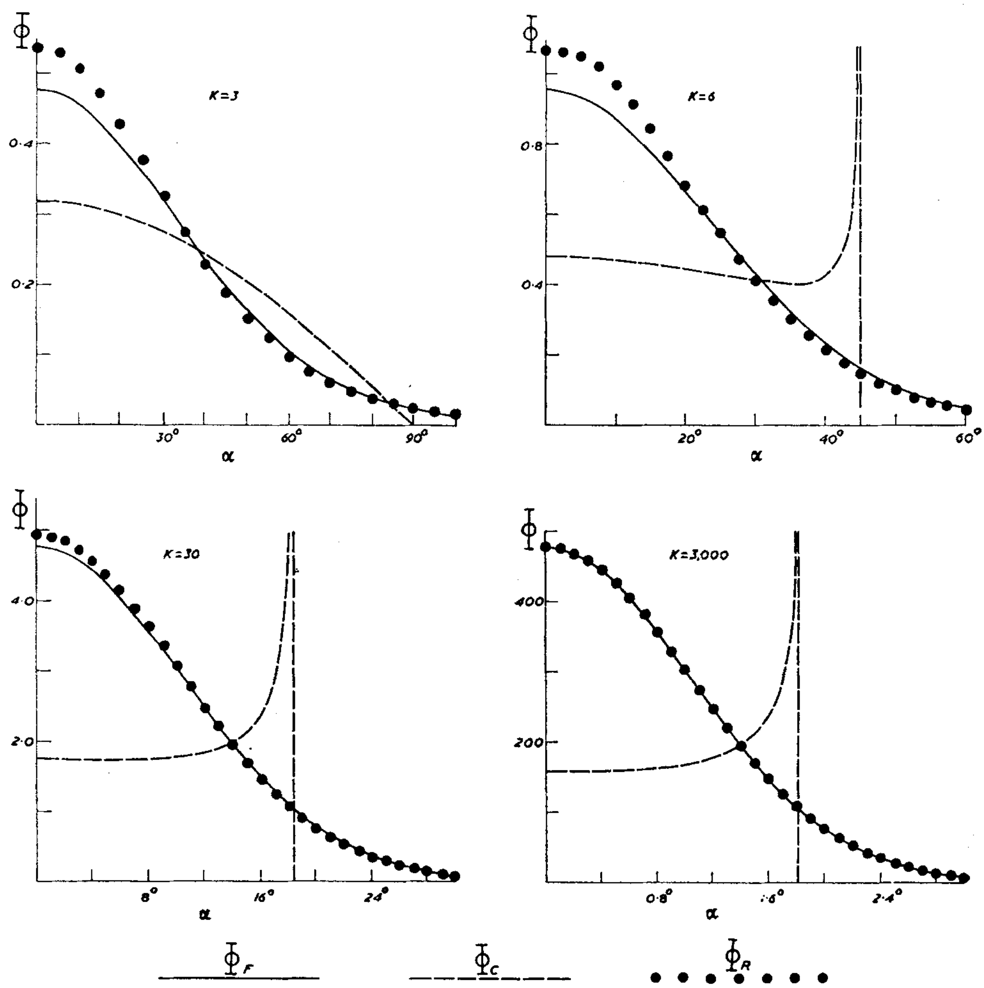

FIG. 2.—Angular probability distribution of Fisher (Φ_F), constant random vector (Φ_C), and sum of equal random vectors (Φ_R) types. For the Φ_C and Φ_R distributions, x_C and x_R are given values such that $\kappa = K_R = K_C$.

To solve the inverse problem of finding M_C and M_0 from the angular dispersion of the resultant vector **J** and its average length, we first find the mean $m\{w(z)\}$ for the probability distribution $w(z)$;

$$m\{w(z)\} = 1 + x_C{}^2/3, \qquad x_C < 1$$

$$m\{w(z)\} = x_C + 1/3x_C, \qquad x_C \geqslant 1. \tag{15}$$

In palaeomagnetic studies values of k or δ and $\bar{J}$, the mean value of the remanent magnetization vector, are generally available from routine calculations. Where the number of vectors used in these calculations is sufficiently large and where this model is physically applicable, k and δ may be used as estimates of K_C and Δ_C, yielding the equations:

$$M_o \cong \bar{J}k/(k+1)$$

$$M_C \cong \bar{J}(3k)^{1/2}/(k+1)\ , \qquad k > 3$$

$$M_o \cong \bar{J}3x_C/(3x_C^2+1)$$

$$M_C \cong \bar{J}3x_C^2/(3x_C^2+1), \qquad k \leqslant 3 \qquad (16)$$

where $x_C = 2k/3\ (k-1)$ for $k \leq 3$. If K_C is substituted for k, these equations apply to the probability distribution and are exact.

4. Sum of random equal vectors

Random magnetization of the constant vector type is probably uncommon among rocks containing ferromagnetic grains with a broad spectrum of high coercive forces, for which the following model may be more appropriate. We assume that **M** is the vector sum of n units of magnetization, each with the same moment **m** but varying randomly in direction. For example, **m** may be the moment of each grain of magnetic mineral and n the number of grains in each specimen. Alternatively, n may be the number of steps in a demagnetization experiment and **m** the magnetization added randomly at each step. As before, the total random component **M** varies randomly in direction from specimen to specimen. Its intensity M also varies from specimen to specimen, and for $n > 6$ the probability distribution $P_R(M)$ is given by the following asymptotic expression due to Rayleigh (1919) and Vincenz & Bruckshaw (1960):

$$p_R(M)\, dM = [4\pi M^2/(2\pi n m^2/3)^{3/2}] \exp(-3M^2/2nm^2)\, dM. \qquad (17)$$

This distribution has a single maximum of $p_R(M_R) = 4/\pi^{\frac{1}{2}}\, e\, M_R$ when $M = m(2n/3)^{\frac{1}{2}} = M_R$. Writing $x_R = M_R/M_o$ and substituting, we obtain

$$p_R(x)dx = (4x^2/\pi^{1/2}x_R^3) \exp\,(-x^2/x_R^2)\, dx \qquad (18)$$

where $p_R(x)\, dx$ is the proportion of vectors **M** with magnitudes between M and $M+dM$. The mean of this distribution is $2x_R/\pi^{1/2}$.

The angular probability distribution for this set of resultant vectors may be found by substituting this function in equation 5 and integrating over the indicated ranges of x with the following result:

$$\Phi_R = \frac{1}{4\pi}\left(\frac{2\cos^2\alpha}{x_R^2}+1\right)\left(1+\operatorname{erf}\frac{\cos\alpha}{x_R}\right)\exp(-\sin^2\alpha/x_R^2)+$$

$$+\frac{1}{2\pi^{3/2}}\frac{\cos\alpha}{x_R}\exp(-1/x_R^2). \qquad (19)$$

The corresponding value of R_R evaluated from equation 7 is

$$R_R = (1-\tfrac{1}{2}x_R^2)\operatorname{erf}\left(\frac{1}{x_R}\right)+\frac{x_R}{\pi^{1/2}}\exp(-1/x_R^2). \qquad (20)$$

For $x_R \ll 1$ the following approximations are useful

$$K_R \cong 2/x_R^2 \tag{21}$$

$$\Delta_R \cong \cos^{-1}(1-\tfrac{1}{2}x_R^2). \tag{22}$$

The error in using these approximations is less than one per cent for $x_R \leq \frac{1}{2}$, corresponding to $K_R \geq 8$ and $\Delta_R \leq 29°$. Exact values of K_R and Δ_R may be found by substituting R_R values in equations 8 and 9. Conversely, if K_R of Δ_R is known, the exact value of x_R may be found from equation 20, noting that $R = (K-1)/K = \cos\Delta$.

The angular dispersion and probability distributions for this model are seen to depend only on the quantity $x_R = m(2n/3)/^{1/2}M_o$. To interpret this result in terms of specimens containing multidomain ferromagnetic grains, let n_V be the number of grains per unit volume and V be the specimen volume. Assume that the random component of each grain is the same, as is $m_o = M_o/n_V V$, the contribution of each grain to the primary component of magnetization $\mathbf{M}_o$ of the specimen. Then $x_R = (m/m_o)(2/3n_V V)^{1/2}$ and $K_R = 3nV(m_o/m)^2$ provided $K_R \gg 1$. Thus where dispersion is entirely due to random magnetization of this type the precision parameter K_R is proportional to specimen volume, and the ratio of the primary to the random component in each grain is equal to the quantity $(K_R/3n_V V)^{1/2}$.

Curves of this angular probability distribution are shown in Figure 2 for a wide variety of angular dispersions. For comparison, curves of the Fisher distribution are shown in each case, and as with the constant–vector distribution the angular dispersions were made equal by selecting x_R to give $K_R = \kappa$. To examine the cumulative effect of small differences between the two curves, the cumulative probability distribution function was found in the same manner as that used in the derivation of equation (13), with the following result:

$$P_R(\alpha_o) = \tfrac{1}{2}\left[\left(1+\operatorname{erf}\frac{1}{x_R}\right)-\left(1+\operatorname{erf}\frac{\cos\alpha_o}{x_R}\right)(\cos\alpha_o)\right]\exp(-\sin^2\alpha_o/x_R^2). \tag{23}$$

The P_F and P_R cumulative distribution functions, although different in analytical expression, yield remarkably close numerical values. For values of $\kappa = K_R = 30$, for example, if $\alpha_o = 25{\cdot}882°$ then $P_F = 0{\cdot}9500$ and $P_R = 0{\cdot}9477$; if $\alpha_o = 32{\cdot}167°$ then $P_F = 0{\cdot}9900$ and $P_R = 0{\cdot}9879$. For higher values of κ, the agreement is closer. Since this degree of fit is about the same as that found by Roberts & Ursell (1960) between the Fisher and Brownian distributions, it appears reasonable to apply their argument that small departures from the assumed distribution will have little effect on statistical deductions based on Fisher's distribution.

The probability distribution $w_R(z)$ is found from equation (4) setting $p(x) = p_R(x)$ with the result:

$$w_R(z) = (z/\pi^{1/2}x_R)\{\exp[-(1-z)^2/x_R^2]-\exp[-(1+z)^2/x_R^2]\}. \tag{24}$$

The mean of this distribution is given by

$$\begin{aligned} m\{w_R(z)\} &= (1+\tfrac{1}{2}x_R^2)\operatorname{erf}(1/x_R)+(x_R/\pi)^{1/2}\exp(-1/x_R^2) \\ &= 1-(1/K_R)+x_R^2\operatorname{erf}(1/x_R). \end{aligned} \tag{25}$$

A convenient approximation valid for small values of x_R is the following:

$$m\{w_R(z)\} \cong 1+1/K_R.$$

Because, as Fisher (1953) has shown, k is the best estimate of κ for a population of the Φ_F type and because the Φ_R and Φ_F distributions are so nearly equal if $K_R = \kappa$, k as evaluated experimentally is also a good estimate of K_R provided this model for angular dispersion is physically applicable. Again using the mean intensity of magnetization $\bar{J}$ of all specimens to estimate the mean for the population, the following equations are valid for $k \gg 1$:

$$M_o \cong \bar{J}k/(k+1)$$
$$\bar{M} \cong \bar{J}(8/\pi k)^{1/2}k/(k+1) \qquad (26)$$

where $\bar{M}$ is the mean intensity of the random vectors, corresponding to $2M_o x_R/\pi^{\frac{1}{2}}$. For small values of k, x_R may be found from equation (20) and substituted in the more exact equations.

5. Experimental test of M calculations

The present method for calculating $\bar{M}$ from experimentally determined values of k and $\bar{J}$ assumes a probability distribution $p(M)$ for the added vectors. However the random magnetization of interest in palaeomagnetic studies may be generated in many ways, and since angular distributions were found to be sensitive to variations in $p(x)$, the question arises of whether the $\bar{M}$ calculations also are. To test this, an analysis was made of some experimental data collected originally to evaluate demagnetization apparatus. A rock specimen was placed in simultaneous rotation about three axes within an alternating magnetic field which was smoothly decreased from a peak value to zero, after which the intensity J_i and the direction cosines λ_i, μ_i, ν_i of the remanent magnetization vector were determined. The experiment was repeated twelve times for a single peak value of the alternating field, the sample being given a different orientation in the apparatus for each experiment. Angular dispersion was developed as a component M_i was added in each experiment to a component M_o which, for physical reasons, is thought to be constant or nearly so for all twelve experiments. Under this assumption and the additional assumption that the added vectors M_i sum to zero, the mean $\bar{M}$ of the added vectors was found from the relation:

$$\bar{M} = (1/N)\sum_{i=1}^{12} [(J_i\lambda_i - \overline{J\lambda})^2 + (J_i\mu_i - \overline{J\mu})^2 + (J_i\nu_i - \overline{J\nu})^2]^{1/2} \qquad (27)$$

where

$$\overline{J\lambda} = (1/N)\sum_{i=1}^{12} J_i\lambda_i$$

etc. In addition the average intensity $\bar{J}$ of the twelve remanent magnetization vectors was calculated, as well as k as defined by equation (2).

Sets of twelve experiments were made on two magnetically different rock specimens under varying experimental conditions in peak fields ranging from 200 to 800 oersteds. The angular dispersion increased at higher fields as M_o decreased and M_i increased, so that k ranged from 9·51 to 21,100; almost all data of interest in palaeomagnetic studies fall within this range. Three conclusions drawn from a detailed analysis of these data are relevant to the present discussion. (1) The added vectors were not of constant length. (2) For some of the experiments, but not for all of them, the direction in which the M_i vectors were added was not entirely random. (3) The added components appear from physical considerations not to have been generated by an ideal three-dimensional random walk of equal steps. Thus neither of the theoretical models exactly describes the way in which this angular dispersion was produced.

To compare these data with theory, values of $\bar{M}/\bar{J}$ evaluated directly (equation 27) are plotted as abscissas in Figure 3, values of $(8/\pi k)^{\frac{1}{2}}\, k/(k+1)$ as

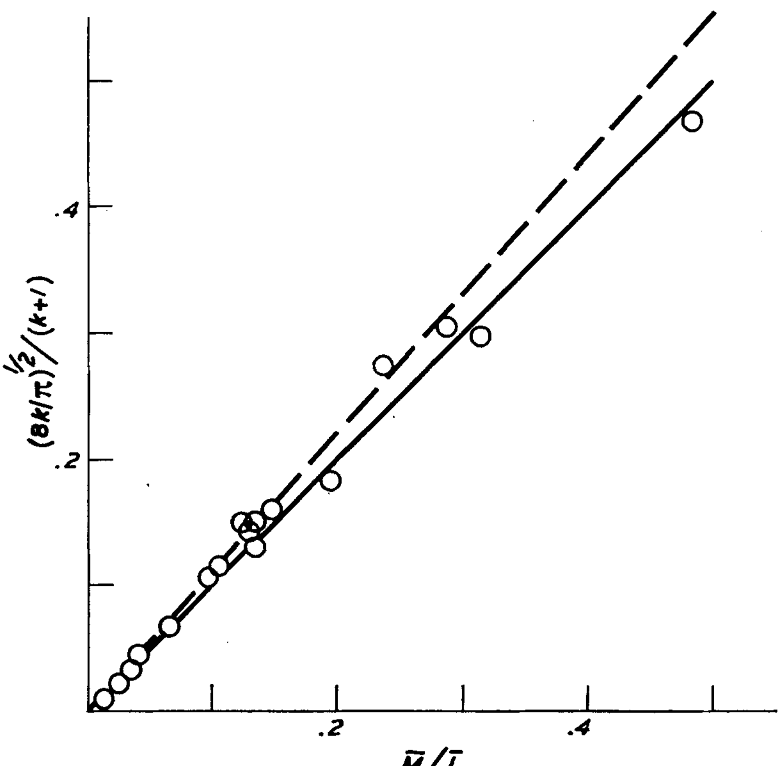

FIG. 3.—Experimental test of method for calculating $\bar{M}/\bar{J}$ from precision parameter k. The solid line corresponds to a Φ_R population, the dashed line to a Φ_C population.

ordinates. If the experimental data consisted of a large sample drawn from a population having the Φ_R and p_R probability distributions, these two quantities would be equal and the data points would lie along the solid line.

The dashed line shows where data drawn from a population of the Φ_C type would lie on this graph. For the Φ_C population $\bar{M}/\bar{J} = (3)^{\frac{1}{2}}k^{\frac{1}{2}}/(k+1) = 1{\cdot}732k^{\frac{1}{2}}/(k+1)$ whereas for the Φ_R population $\bar{M}/\bar{J} = (8/\pi)^{\frac{1}{2}}k^{\frac{1}{2}}/(k+1) = 1{\cdot}596k^{\frac{1}{2}}/(k+1)$. The effect of a $p(x)$ distribution with a smaller first moment may be seen from the following calculation for a uniform distribution of $p(x)$ over a band of width $2\Delta x$ with centre at x_B. It can be shown that $k = 3/(x_B{}^2+\Delta x^2/3)$ and, as for the other cases, $\bar{J} = M_o(k+1)/k$. If the band extends from O to $2x_B$, then $\bar{M} = x_B M_o$ and $\bar{M}/\bar{J} = (3/2)k^{\frac{1}{2}}/(k+1)$. Thus the effect of concentrating the x distribution in a line spectrum leads to an increase of $\bar{M}/\bar{J}$ of only 8·6 per cent over that for the Φ_R distribution, whereas concentration of x in a uniform band at $x =$ O decreases $\bar{M}/\bar{J}$ by only 6 per cent.

In view of the fact that each datum point (Figure 3) is based on only twelve experiments, most points lie remarkably close to the line for the Φ_R distribution, although many of them would be equally consistent with the Φ_C or the band distribution. These results suggest that calculations of the mean length of random vectors are not very sensitive to moderate departures from the theoretical models.

6. Discussion

The angular probability distribution generated by adding randomly oriented vectors to constant vectors was shown to depend on the probability distribution of the lengths of the added vectors. If these are generated by a three-dimensional random walk of a large number of small steps, a one-parameter length distribution results. This in turn produces a one-parameter angular probability distribution characterized by the mode x_R of the length distribution, and this angular distribution is remarkably similar to the one-parameter distribution of Fisher. The two distributions have nearly equal numerical values over almost the entire range of angular dispersions encountered in palaeomagnetic research.

The significance of the parameter κ of the Fisher distribution is that the quantity $(2/\kappa)$ is the population variance, $(2/k)^{\frac{1}{2}}$ being an estimate of the sample angular standard deviation (Fisher 1953, Roberts & Ursell 1960, Creer *et al.* 1959). In the present model, $(2/k)^{\frac{1}{2}}$ has the additional significance of being an estimate of x_R, the mode of the distribution of the lengths of the added vectors. Where angular dispersion is due to random magnetization of multidomain grains, the additional relationship exists that $(2/k)^{\frac{1}{2}} = (3/2)^{\frac{1}{2}}(m/m_o)/T^{\frac{1}{2}}$ where (m/m_o) is the ratio of the random to the constant component of magnetization of each grain and T is the total number of ferromagnetic grains in each specimen.

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